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Vibration Control of Spatial Structures using Spatially Distributed MTMDs

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Synopsis

A new vibration control design method applying TMD (Tuned Mass Damper) to spatial structures is proposed in this paper. In general, spatial structures have various vibration modes involving rather high-order modes and those natural frequencies are closely spaced. Then, in order to control those modes, a spatially distributed MTMDs (Multiple TMD) method is proposed in this paper. Firstly we explain details of the proposed method on the spatial arrangement of MTMDs, the set-up procedure of design parameters and the standard of applicable limit. Next we apply the proposed method to a single layer lattice shell and compared its control performance to the conventional single TMD method.

KEYWORDS: Vibration control, Spatial structures, TMD, MTMD, Acceleration response spectrum

1. Introduction

Because spatial structures have superior load transmitting performance, it has been generally recognized that they are lighter and therefore safer in even strong earthquakes than general beam structures. However, results of investigations following recent strong earthquake disasters have shown that people often could not use these structures for emergency shelters because of slight damage to the structures themselves as well as dropping of ceilings and suspended objects [1),2)].

Then a new vibration control method applying TMD (Tuned Mass Damper) to spatial structures is proposed. We select TMD for vibration control devices because they have been widely applied to large-span light-weight structures [3)] and are considered to be superior especially in application to complex-shaped large-span roof structures because they can be installed with only one support point.

Spatial structures have different vibration properties than general building structures. In this study, we focused on the following two properties:

- Various vibration modes involving rather high-order vibration modes
- Closely spaced natural frequencies of those vibration modes

In order to control those modes, we have been proposed a spatially distributed MTMDs (Multiple TMDs) method shown in Figure 1 [4)]. Because size of spatially distributed MTMDs are small, it can be considered to apply them inside structural members or joints.

The basic MTMD theory to control one vibration mode was already proposed by Professor Yozo Fujino of the University of Tokyo [5)-7)]. Conventional single TMD consists of one mass tuned to a natural frequency of a controlling mode. On the other hand, the MTMD method shown in Figure 2 consists of several small-sized MTMDs with different natural frequencies distributed over an optimal bandwidth around a natural frequency of a controlling mode. In other studies about MTMD method, Takeru Igusa proposed design formulas to minimize a mean square value of the structural response under white noise loading [8),9)]. A.Kareem and S.Kline executed parametric studies, e.g. total bandwidth of MTMDs, tuning ratio of each MTMD and number of MTMDs, under random loading [10)]. G.Chen and J.Wu proposed a sequential procedure for practical design and arrangement of MTMDs of building structures subjected to seismic motion [11)].

We have been studied applicability of the spatially distributed MTMDs to control plural modes of large-span structures with closely spaced natural frequencies using a rectangular plate model [4)]. From the analytical results, the following characteristic properties of the proposed method were obtained:

- In the spatial domain, responses of overall structures are even.
- In the frequency domain, responses inside the MTMD bandwidth are even.

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In conclusion, we see that the effect of vibration control of the proposed method for spatial structures was superior to the single TMD method with the same total weight. But in the papers [4], we only applied the existed MTMD method proposed for controlling one mode by adjusting MTMD bandwidth intuitively. And we did not propose a new design method theoretically for controlling plural modes.

Therefore, in this paper, the spatial arrangement and the set-up procedure of design parameters of MTMDs for controlling plural modes are newly proposed and the standard of applicable limit is also proposed.

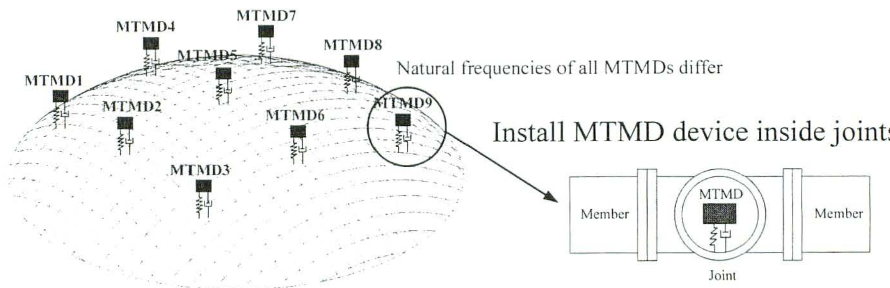


Figure 1. Spatially Distributed MTMD method

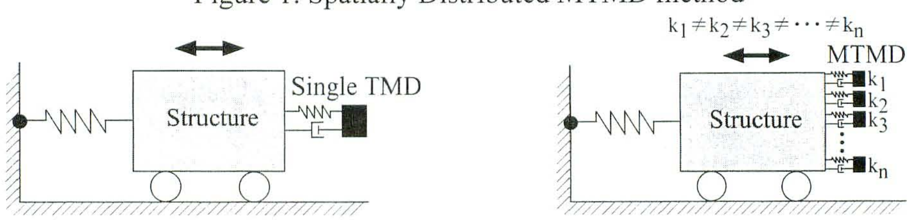


Figure 2. Single TMD and MTMD method

2. Design Method of Spatially Distributed MTMDs

The proposed design method is characterized by controlling various vibration modes together using robust property of MTMD method against variation of natural frequencies of structures. In this chapter, details of the proposed method about the spatial arrangement and the set-up procedure of design parameters of MTMDs are described.

2.1 Spatial Arrangement of MTMDs

In the conventional single TMD method, TMDs are arranged at largest amplitude points (antinodes) of each controlling mode and design parameters are set up for each mode individually. On the other hand, design parameters of MTMDs are set up to couple strongly all MTMDs and structure. Therefore, the spatial arrangement of MTMDs should be determined to couple strongly all MTMDs and structure. Then, we arrange MTMDs to maximum amplitude points of the superposed shape of controlling modes. Moreover, in order to control the overall structures in the well-balanced manner, MTMDs are dispersed arranged. Figure3 compares the spatial arrangement of (a) the single TMD and (b) the spatially distributed MTMDs methods.

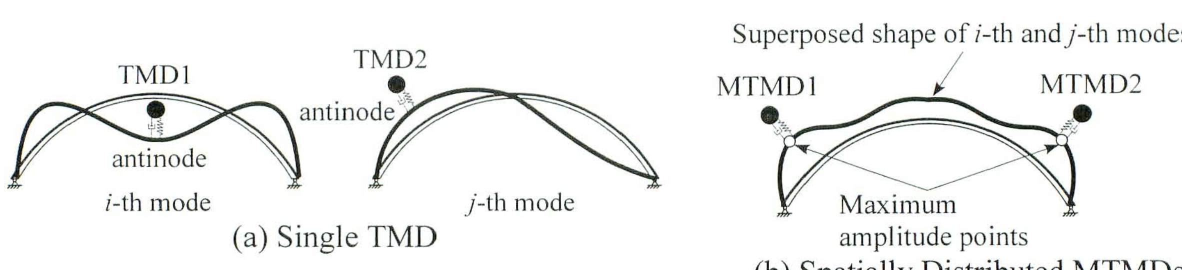


Figure 3. TMD and MTMD arrangement

2.2 Design Parameters

Firstly, we summarize the set-up procedure of MTMD for controlling one mode proposed by the existed study[6]. Mass of all MTMDs and spaces between natural circular frequencies of each MTMD are all equal. Figure 4 shows the distribution of each MTMD on the natural circular frequency axis.

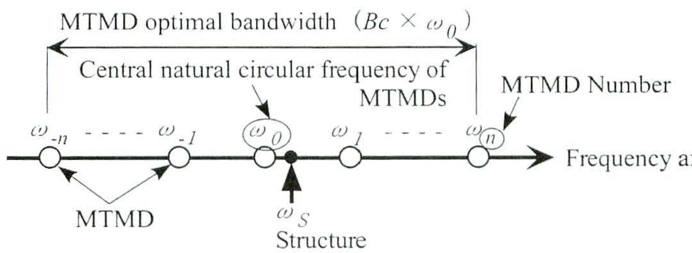


Figure 4. Distribution of natural circular frequencies of MTMDs

Number of MTMDs is N (=2n+1). n shows MTMD number. The relation between central natural circular frequency ω_0 of all MTMDs and natural circular frequency ω_j of other MTMDs is shown by equation (1). The relation between ω_0 and natural circular frequency ω_s of the structure to be controlled is shown by equation (2).

$$\omega_j = \omega_0(1 + j\beta) \quad (j = -n \sim n) \tag{1}$$

$$\omega_0 = \omega_s(1 + \beta_0) \tag{2}$$

where β and β_0 show non-dimensional frequency spaces between each MTMD and between ω_0 and ω_s, respectively. Non-dimensional MTMD bandwidth B is defined as

$$B = (\omega_n - \omega_{-n}) / \omega_s \tag{3}$$

The relation between β and B is expressed as

$$\beta = B/(N-1) = B/(2n) \tag{4}$$

Central natural circular frequency ω_0 of MTMDs is takes as

$$\omega_0 = \omega_s \Big/ \sqrt{1 + \mu_{total}} \tag{5}$$

where μ_{total} is total mass ratio of MTMDs. The optimal bandwidth B_c is defined as

$$B_c = \sqrt{(\mu_{total} T)/2} \tag{6}$$

where $T = \gamma + \log(N)$ ($\gamma = 0.57721$: Euler's constant).

The optimal bandwidth B_c is determined by the criterion that all MTMDs were strongly coupled to the structure for all modes in order to control the structure in the well-balanced manner around frequency range of external force.

The damping ratio of MTMD is determined as

$$\xi_T = \sqrt{3}\beta / \pi \tag{7}$$

When non-dimensional MTMD bandwidth B is determined from equation (3), non-dimensional frequency space β is determined. Then, natural circular frequency ω_j and damping ratio ξ_T of each MTMD are determined from equations (1) and (7), respectively.

In the case of controlling plural modes, MTMD bandwidth B is to be determined by involving all the optimal bandwidth B_c of controlling modes. Then, the structure can be controlled well in the well-balanced manner around frequency range of controlling modes, because all MTMDs are strongly coupled to the structure.

Figure 5 shows MTMD bandwidth B in the case of controlling i-th, j-th and k-th vibration modes.

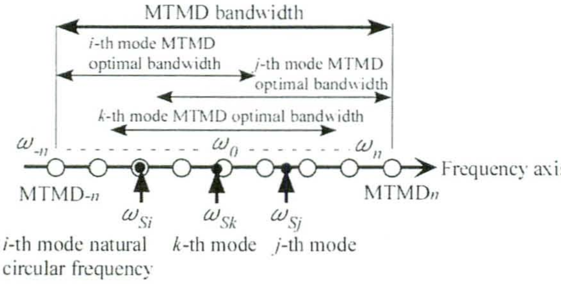


Figure 5. MTMD bandwidth to control plural modes

2.3 Standard of Applicable Limit

From the existed study [6], additional damping ξ_{eq} for harmonic oscillations is defined as

$$\xi_{eq} = \frac{\mu}{2\beta}\tan^{-1}\left(\frac{n\beta}{\xi_T}\right) \approx \frac{\rho}{2}\tan^{-1}\left(\frac{B}{2\xi_T}\right) \tag{8}$$

where μ is mass ratio of one MTMD and $n(=\frac{N-1}{2})$ is MTMD number. Mass density ρ of MTMDs is expressed as

$$\rho = \mu_{total} / B \approx \mu / \beta \tag{9}$$

Because the damping ratio of MTMD is gives as equation (7),

$$\frac{B}{\xi_T} = \frac{(N-1)\pi}{\sqrt{3}} \tag{10}$$

When number of MTMDs N is equal, equation (10) have the same value and from equation (8) additional damping ξ_{eq} is in proportion to mass density ρ.

Figure 6 shows MTMD bandwidth in the case that a natural frequency space between *i*-th and *j*-th modes to be controlled is narrow and the optimal bandwidths of both modes are overlapped. In this case, mass density is higher than the case with applying the optimal bandwidths of both modes, and as a result control performance becomes high.

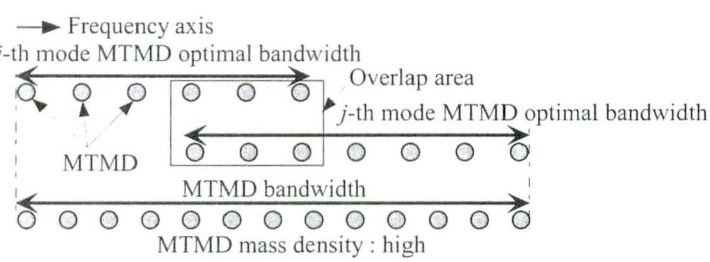


Figure 6. In the case of the optimal bandwidths to be overlapped

On the other hand, in the case that a natural frequency space between *i*-th and *j*-th modes is wide and the optimal bandwidths of both modes are not overlapped, mass density is smaller than the case with applying the optimal bandwidths, and as a result control performance becomes small.

From the above mentioned, we may conclude that the standard of applicable limit in the frequency domain is the case that the optimal bandwidths are bordered shown in Figure 7. Therefore, when each optimal bandwidth does not so differs, the standard of applicable limit of MTMD bandwidth can be considered twice the optimal bandwidth.

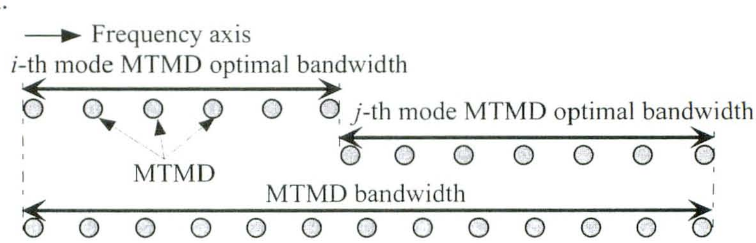


Figure 7. In the case of the optimal bandwidths to be bordered

4. Numerical Analysis

In order to study control performance of the proposed method, numerical analyses using a single layer lattice shell model are executed. We adopt two recorded seismic waves for external force.

4.1 Analytical Model

The dimension of plan, rise and rise-span ratio is 50×50m, 12.5m and 0.25, respectively, shown in Figure 8. The material property of structural elements is steel and steel pipe 318.5×9.0mm is used for all elements. The joints between all elements are rigidly connected. The boundaries of both gable sides are pinned supports and those of both straight-line ends are free. Only structural dead load is considered. We use beam elements possessing axial, bending and torsion rigidities.

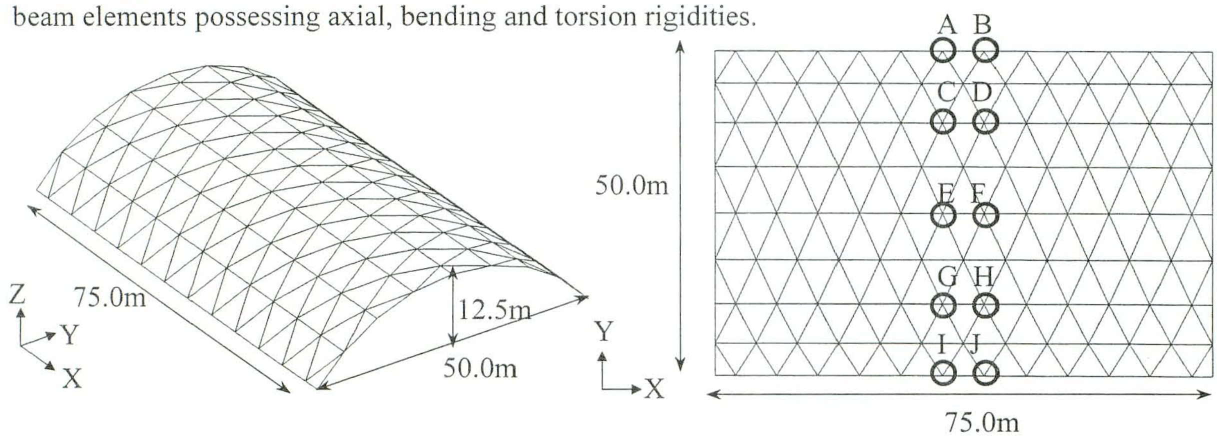


Figure 8. Analytical model

4.2 Modal Analysis

In order to grasp basic vibration property of the model and design MTMDs, a linear modal analysis is executed. Table 1 shows natural frequencies, oscillated direction and positions of antinodes of chosen vibration modes having less than 5.0Hz natural frequencies and more than 0.005 effective mass ratios. Shapes of 2nd and 16th modes are shown in Figure 9.

Order of modes	Natural frequencies (Hz)	Oscillated direction (Figure8)	Effective mass ratio	Positions of antinodes (Figure8)
1	0.9071	Z	0.09428	A,B,I,J
2	0.9467	Y	0.09848	A,B,I,J
5	2.308	Y	0.006292	A,B,I,J
6	2.314	Z	0.006263	A,B,I,J
7	2.500	Y	0.06522	C,D,G,H
8	2.534	Z	0.008794	A,B,I,J
16	4.445	Z	0.6358	E,F

Table 1: Results of modal analysis

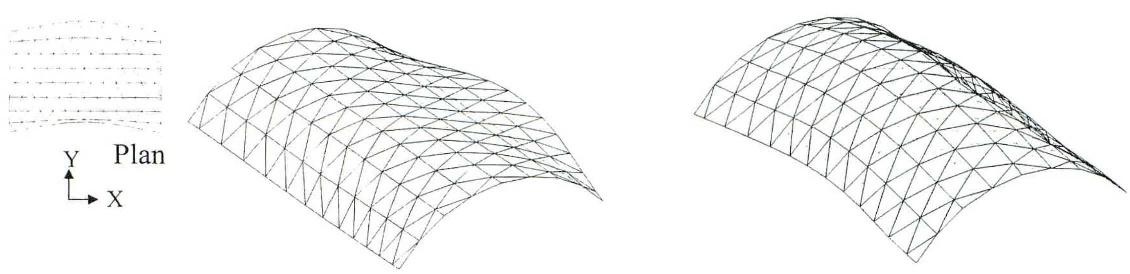


Figure 9. Modal shape ((a)Left:2nd mode, (b)Right:16th mode)

4.3 Design of MTMDs and Single TMD

Because we apply seismic waves for external force and consider spectrum distribution of general seismic waves, we choose seven modes shown in Table 1 for controlling modes. Moreover, as described in the chapter 2.3, the proposed method is most effective in the case that natural frequencies of controlling modes are close spaced, seven controlling modes are divided into three groups according to distribution of natural frequencies shown in Table 1.

(1) 1st and 2nd modes : plural modes control
(2) From 5th to 8th modes : plural modes control
(3) 16th mode : single mode control

Total mass ratio of MTMDs to the structure is 3.0%. Each mass of MTMD is divided in proportion to sum of effective mass of three groups.

The spatial arrangement of MTMDs is determined by using the superposed shape of three controlling groups following the method described in chapter 2.1.

Determined design parameters and the spatial arrangements with sixteen MTMDs are shown in Table2.

MTMD Number	Control group	Position of MTMD (Figure8)	Mass (Kg)	Spring constant (N/m)	Damping coefficient (N-sec/m)
1	1	A	249.92	6,141.1	111.33
2	1	B	249.92	7,334.4	121.67
3	1	I	249.92	8,634.6	132.00
4	1	J	249.92	10,039	142.34
5	2	A	56.120	9,984.1	25.539
6	2	B	56.120	10,689	26.425
7	2	I	56.120	11,418	27.312
8	2	J	56.120	12,171	28.198
9	2	C	56.120	12,948	29.084
10	2	D	56.120	13,749	29.970
11	2	G	56.120	14,574	30.856
12	2	H	56.120	15,424	31.743
13	3	E	824.34	416,580	2,224.9
14	3	F	824.34	532,070	2,514.5
15	3	E	824.34	661,660	2,804.0
16	3	F	824.34	805,370	3,093.6

Table 2 : Design parameters and spatial arrangements with 16 MTMDs

To compare control performance of the proposed method to the conventional single TMD method, we apply single TMD to 1st and 2nd modes respectively according to equations (11) and (12) by the theory of Den Hartog [12]. Total mass of single TMD is equal to MTMDs.

$$\gamma_{opt} = \frac{1}{1+\mu} \tag{11}$$

$$\xi_{opt} = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{12}$$

where γ_{opt} , ξ_{opt} are optimum tuning ratio and optimum damping ratio. μ is mass ratio of one TMD to equivalent mass of the structure. Determined design parameters and the spatial arrangements of two single TMDs are shown in Table3.

Control mode	Position (Figure8)	Mass (Kg)	Spring constant (N/m)	Damping coefficient (N-sec/m)
1	A	2,373.0	66,243	6,442.5
2	A	2,373.0	71,577	6,898.8

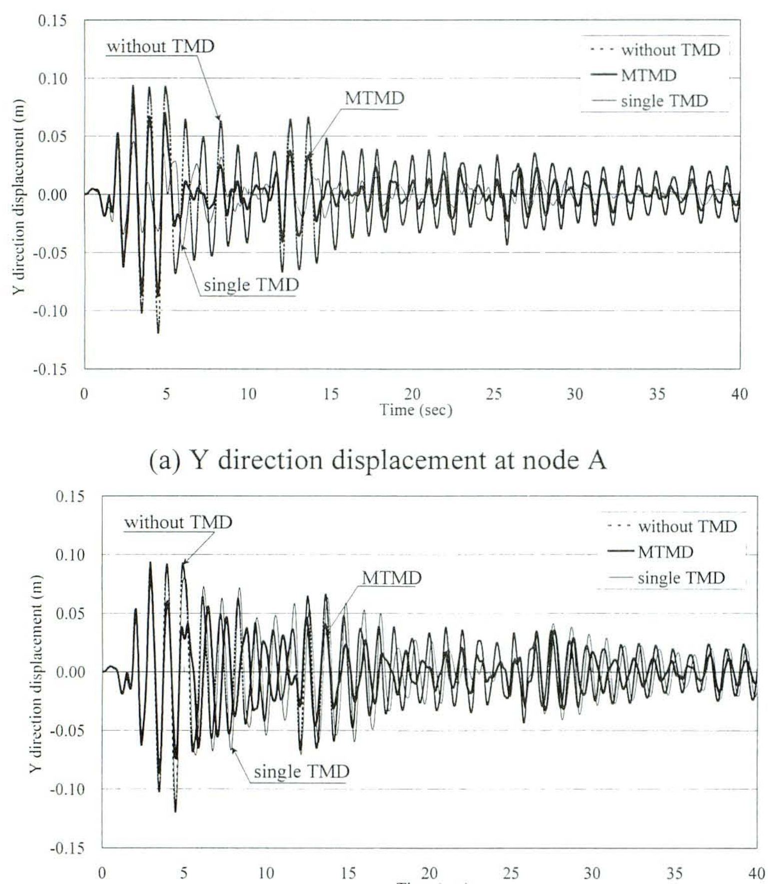
Table 3 : Design parameters and spatial arrangements with two single TMDs

4.4 Earthquake Response Analysis

To study control performance subjected to seismic loads, earthquake response analyses are executed. We use direct transient analytical method and its interval of integral calculus time is set to 0.01 second. Stiffness proportional damping 2.0% at the natural frequency of 1st mode is used. We apply two recorded seismic waves : El Centro 1940 NS wave (Y direction), UD (Z direction) and Taft 1952 EW wave (Y direction), UD (Z direction). And total loading time is 40.0 seconds.

(1) Comparison of time history of response displacement

Figure10 shows Y direction time history of response displacements at node A and node I of antinodes of 2nd mode subjected to El centro 1940 NS wave for without TMD, single TMD and MTMD cases. We may see that a certain time is needed to emerge control performance of TMD and MTMD. And it can be recognized that single TMD is effective at node A with TMD position, but at node I with no TMD position displacements of single TMD are larger than without TMD case at several time points. MTMD is very effective for both nodes. Characteristics described above were also recognized for other seismic waves.



(a) Y direction displacement at node A

(b) Y direction displacement at node I

Figure 10. Time history of response displacement (El Centro 1940 NS)

(2) Comparison of response displacements with all nodes

In order to grasp characteristics of response displacements with the overall structure, we compare about sum of mean square value on response displacements with all nodes during loading time. Figure 11 and Figure 12 shows comparisons of sum of mean square value on response displacements with Y and Z directions subjected to El Centro 1940 and Taft 1952 waves. We may see that MTMD is very effective compared to single TMD for controlling the overall structure.

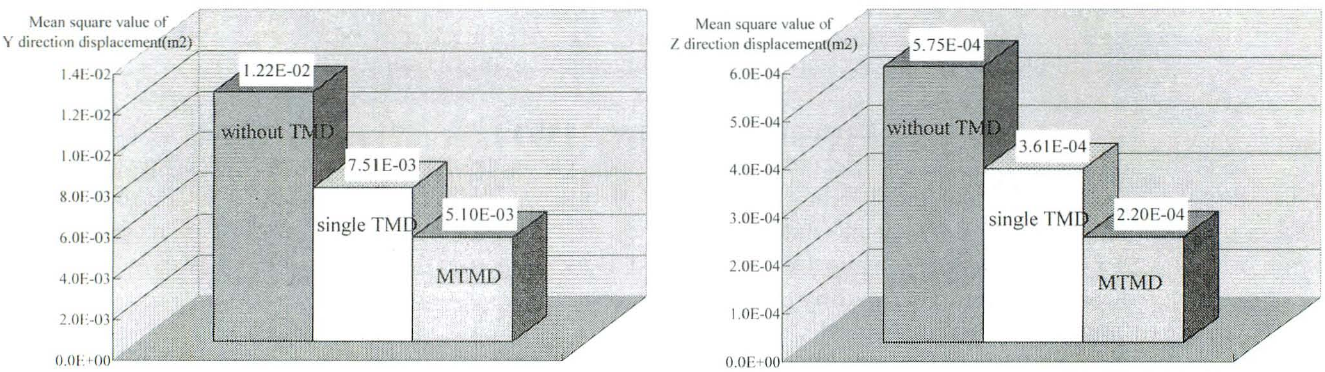


Figure 11. Comparison in the case of El Centro 1940 wave ((a)Left:NS, (b)Right:UD)

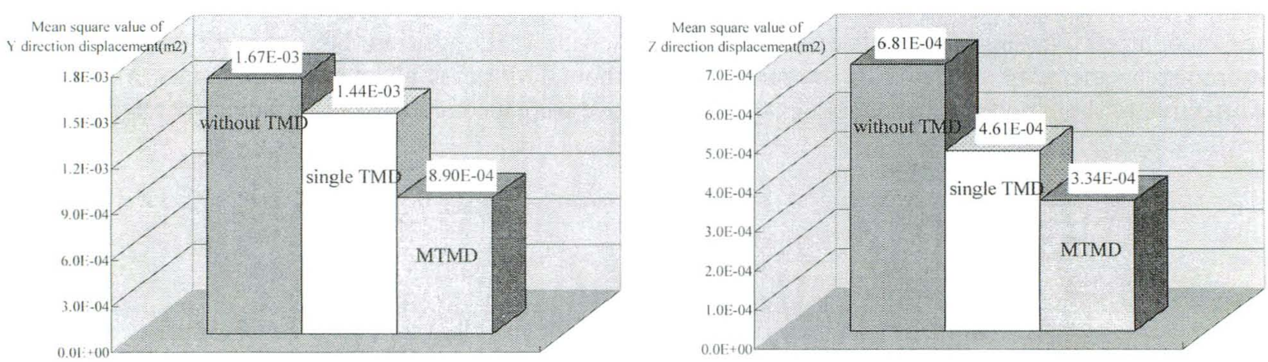


Figure 12. Comparison in the case of Taft 1952 wave ((a)Left:EW, (b)Right:UD)

Figure 13 and 14 show the distribution of mean square value on response displacements with El Centro 1940 and Taft 1952 loadings, respectively. The same scale of displacement is used for all figures. Single TMD is very effective at the node with TMD position, but responses over all the structure are uneven. On the other hand, MTMD is very effective over the structure on the whole.

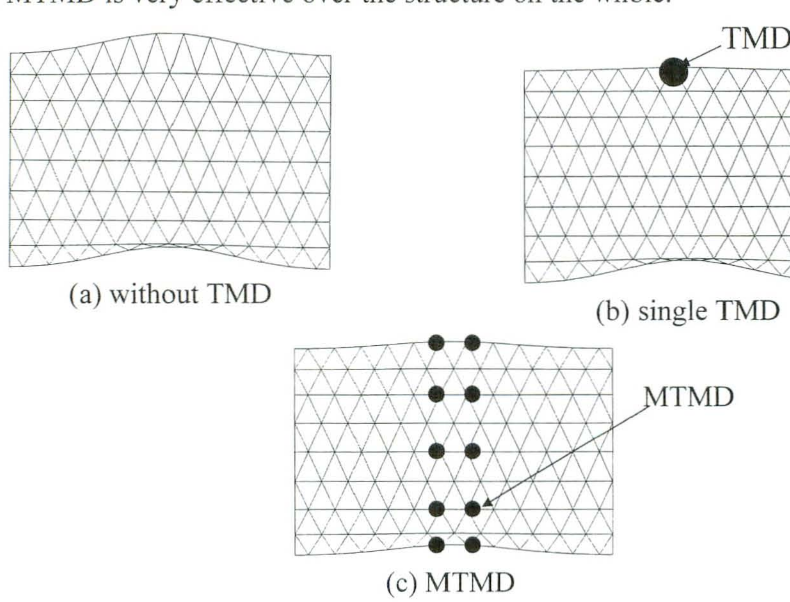
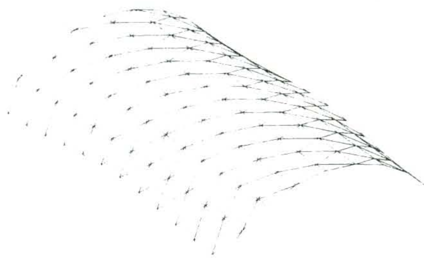


Figure 13. Comparison of mean square value with Y direction response displacement
(El Centro 1940 NS)

(a) without TMD

(b) single TMD



(c) MTMD

Figure 14. Comparison of mean square value with Z direction response displacement (El Centro 1940 UD)

Comparing Figure 11(a) and 12(a), the effect of the vibration control with El Centro 1940 NS wave is larger than Taft 1952 EW wave. Then, the acceleration response spectra of two waves with damping ratio 2.0% are compared in Figure 15. Figure shows the frequencies of 2nd and 7th controlling modes also. It can be recognized that in the case of Taft 1952 EW wave the acceleration response spectrum of controlling modes are small compared to other frequencies.

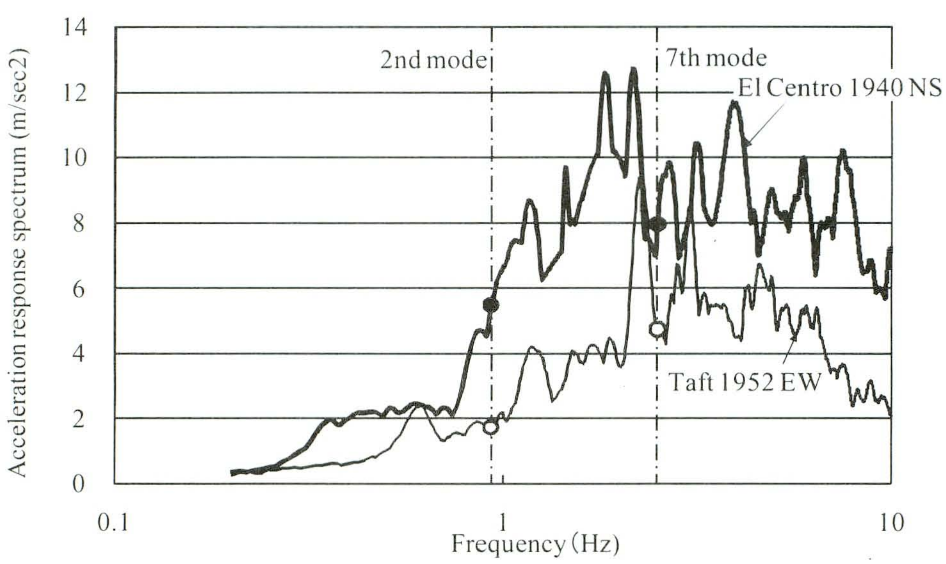


Figure 15. Comparison of acceleration response spectra of two waves

5. Conclusions

This paper proposed the spatially distributed MTMDs method to control large-span structures. This paper extended the existed MTMD method for controlling one mode to control plural modes with closely spaced natural frequencies and proposed the spatial arrangement of MTMDs using the superposed shape of controlling modes. Moreover, we proposed the standard of applicable limit in the frequency domain. Lastly, the control performance of the proposed method to seismic load using the 50 m span single layer lattice shell model is studied. From the analytical results, it could be concluded that the proposed method was superior to the single MTMD method controlling over the structure on the whole.

Acknowledgments

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